

Constraining the topology of the Universe using the polarised CMB maps

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ABSTRACT

We study the possibility for constraining the topology of the Universe by means of the matched circles statistic applied to polarised cosmic microwave background (CMB) anisotropy maps. The advantages of using the CMB polarisation maps in studies of the topology over simply analysing the temperature data as has been done to-date are clearly demonstrated. We test our algorithm to search for pairs of matched circles on simulated CMB maps for a universe with the topology of 3-torus. It is found that the noise levels of both Planck and next generation CMB experiments data are no longer prohibitive and should be low enough to enable the use of the polarisation maps for such studies. For such experiments the minimum radius of the back-to-back matched circles which can be detected are determined. We also showed that the polarisation generated after reionisation does not have an impact on detectability of the matched circles.

Key words: methods: data analysis — cosmic background radiation — cosmology: observations

1 INTRODUCTION

According to General Relativity the local properties of spacetime geometry are described by the Einstein gravitational field equations. However, they do not specify the global spatial geometry of the Universe, i.e. its topology. This can only be constrained by observations. The question of a possible multiply connected topology of our cosmic space was already raised by Schwarzschild (1900) before the formulation of General Relativity as well by de Sitter (1917) and Friedmann (1924) immediately following the formulation of the theory. However, because of a lack of observational data probing the largest scales of the Universe, the standard cosmological model was tacitly assumed to comprise a universe possessing a simply-connected topology. One of the first attempts to constraining the Universe's topology was undertaken utilising the distribution of quasars by Fang & Sato (1985). However, the first measurements of the CMB anisotropy led to a significant improvement in constraints (Stevens, Scott, & Silk 1993; de Oliveira-Costa & Smoot 1995; de Oliveira-Costa, Smoot, & Starobinsky 1996). Such anisotropy maps survey scales comparable to the hori-

zon of the observable Universe and therefore provide the best chance for detection of the signatures of multi-connectedness.

Moreover, in the last decade the study of topology has also drawn considerable attention due to various anomalies observed on the largest angular scales in the *WMAP* data. Some of these anomalies, such as the suppression of the quadrupole moment (Bennett et al. 2003) and an alignment between the preferred axes of the quadrupole and the octopole (de Oliveira-Costa et al. 2004; Copi et al. 2004; Bielewicz et al. 2005), may be explained as effects caused by a multi-connected topology.

Current constraints on topology were established by searching for the signatures of multi-connectedness in CMB sky maps either in the spherical harmonic domain (de Oliveira-Costa et al. 2004; Kunz et al. 2006, 2008) or directly in pixel space (Cornish et al. 2004; Aurich, Lustig, & Steiner 2005, 2006; Then 2006; Key et al. 2007; Bielewicz & Banday 2011). In the latter case, one of the most sensitive methods is the search for matched circles in the temperature anisotropy patterns (Cornish, Spergel, & Starkman 1998). By means of this method as applied to the *WMAP* maps, a large class of topologies has been ruled out. However, the method is not inherently limited to temperature anisotropy studies.

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It can also be applied to the CMB polarisation data. In this work, we investigate such an application of the method. We test it on simulated CMB maps for a flat universe with the topology of a 3-torus, and explicitly consider the possibility for the detection of matching circle pairs for data with an angular resolution and noise level characteristic of the Planck (Planck Collaboration 2005) and CORe (CORe Collaboration 2011) data. The latter will be treated as a reference mission for the next generation of CMB experiments.

The paper is organised as follows. In Section 2 we briefly review some basic properties of the CMB temperature and polarisation anisotropies. A description of the simulations for a universe with multi-connected topology used to test the reliability of our codes is presented in Section 3. Section 4 is devoted to a description of the statistic adopted in our studies. The disadvantages of the search of the matched circles in temperature maps are presented in Section 5 and in Section 6 the application of the method to simulated polarisation maps is shown. In the last section we present our conclusions.

2 CMB ANISOTROPIES

In this section we provide a brief resume on the properties of CMB temperature and polarisation anisotropy.

2.1 Temperature anisotropies

The CMB is observed as blackbody radiation with temperature $T_0 = 2.725 \pm 0.002$ K (Mather et al. 1999). After accounting for the peculiar motion of the Earth with respect to the CMB rest frame, the radiation is seen to exhibit tiny temperature fluctuations of order $\Delta T/T_0 \sim 10^{-5}$. In the Newtonian gauge, the temperature fluctuations in a given direction of the sky, \hat{n} , can be written as

$$\frac{\Delta T}{T_0}(\hat{n}) = \left(\frac{\delta_\gamma}{4} + \psi \right)_{|\tau_r} + \hat{n} \cdot \mathbf{v}_{|\tau_r} + \int_0^{\tau_0} e^{-\kappa(\tau)} (\dot{\psi} + \dot{\phi}) d\tau, \quad (1)$$

where δ_γ is the fractional energy density fluctuations in the CMB, \mathbf{v} is the baryon velocity, ϕ and ψ are the two gravitational potentials in terms of which the metric fluctuations are given by $ds^2 = a^2(\tau) [-(1+2\psi)d\tau^2 + (1-2\phi)dx^2]$, and the optical depth $\kappa(\tau) = \int_{\tau_r}^{\tau_0} a(\tau') \sigma_T n_e x_e d\tau'$. τ is conformal time with subscripts 0 and r denoting the present time and the time of recombination, respectively.

The different terms in equation (1) describe different physical effects. The first term corresponds to photon energy density fluctuations and gravitational potential redshift (so called the Sachs-Wolfe effect, SW), the second to the Doppler effect caused by baryon motions at recombination and the third is the so-called the integrated Sachs-Wolfe effect (ISW) caused by the variation of the gravitational potential with time. The equation does not include the terms coming from other secondary effects such as the Sunayev-Zeldovich and gravitational lensing effects. However, they are important for angular scales smaller than the scales of interests in this paper ($\sim 20'$).

2.2 Polarisation anisotropies

Linear polarisation in the CMB is generated by Thomson scattering of photons by electrons either at the moment of recombination, when photons decouple from the primordial plasma, or during reionisation when, due to partial ionisation of the matter, free electrons reappear again in the Universe. The former produces a pattern of polarisation anisotropy at small angular scales while the latter generates structure on large scales, corresponding to the angular scale of the Universe at the moment of scattering. The small-scale polarisation anisotropy is considered to be a snapshot of the last scattering surface (LSS).

To generate polarisation the radiation incident on the electrons needs to have a quadrupole moment. It can be produced either by scalar or tensor perturbations. Depending on this, the polarisation pattern of the sky will manifest different types of parity. The polarisation modes generated by the former will show “electric-type” parity while the modes generated by the latter – “magnetic-type” parity (Zaldarriaga & Seljak 1997). As is common, we will refer to these two types of polarisation components as the E-modes and B-modes, respectively. We will not consider in this paper the impact of the gravitational lensing effect on the primordial polarisation since it is not important for the angular scales of interest here.

In the case of scalar perturbations, gradients in the velocity field, $\partial_i \mathbf{v}_j$, of the photon-baryon fluid on the LSS lead to a quadrupole component of the temperature seen by the scatterer $\Delta T(\hat{n}') \approx \hat{n}'_i \hat{n}'_j \partial_i \mathbf{v}_j$, which, through Thomson scattering, is converted into polarisation. Then, the polarisation expressed in terms of the Stokes parameters Q and U is given by (Seljak & Zaldarriaga 1998)

$$(Q+iU)(\hat{n}) \propto \sigma_T \int d\hat{n}' (\mathbf{m} \cdot \hat{n}')^2 \Delta T(\hat{n}') \propto \Delta\tau_r \mathbf{m}^i \mathbf{m}^j \partial_i \mathbf{v}_j|_{\tau_r}, \quad (2)$$

where σ_T is the Thomson cross section, $\Delta\tau_r$ is the width of the LSS, $\mathbf{m} = \hat{e}_1 + i\hat{e}_2$ and \hat{e}_1, \hat{e}_2 are directions perpendicular to \hat{n} that are used to define the Stokes parameters. Tensor $\mathbf{m}^i \mathbf{m}^j$ assures that the quantities $Q \pm iU$ transform under rotations of (\hat{e}_1, \hat{e}_2) by an angle γ as spin 2 quantities i.e. $(Q \pm iU)'(\hat{n}) = e^{\mp 2i\gamma} (Q \pm iU)(\hat{n})$.

The Stokes parameters are direct measurables. However, because of their spin 2 nature, they are not convenient for the analysis of the polarisation field, for which the use of spin 0 maps of the E-modes and B-modes is preferred and allows easy discrimination of the type of perturbations generating the polarisation anisotropy. Such quantities can be related to the Stokes parameters by means of spin-raising $\bar{\partial}$ and spin-lowering ∂ differential operators (Zaldarriaga & Seljak 1997), i.e.

$$\begin{aligned} \tilde{E}(\hat{n}) &= -\frac{1}{2} [\bar{\partial}\bar{\partial}(Q+iU)(\hat{n}) + \partial\partial(Q-iU)(\hat{n})], \\ \tilde{B}(\hat{n}) &= \frac{i}{2} [\bar{\partial}\bar{\partial}(Q+iU)(\hat{n}) - \partial\partial(Q-iU)(\hat{n})]. \end{aligned} \quad (3)$$

It is worth noticing that the former equation corresponds to the divergence operator, the latter to the rotation operator of the spin-2 fields on the sphere. Thus, these relations can

be seen as analogs of Gauss's law and Ampere's law¹ respectively, in electrodynamics, and the E-mode and B-mode maps as analogs of the electric charges and the current densities. This explains why these two components of the polarisation field are often called the “electric” and “magnetic” modes.

To avoid the direct differentiation of the $Q \pm iU$ maps decomposition can proceed in the spin 2 spherical harmonics basis $\pm_2 Y_{\ell m}$ i.e. $(Q \pm iU)(\hat{n}) = \sum_{\ell, m} a_{\pm 2, \ell m} \pm_2 Y_{\ell m}(\hat{n})$. Then, the relations (3) expressed by the appropriate spherical harmonics coefficients take the form

$$a_{\ell m}^E = -\frac{a_{2, \ell m} + a_{-2, \ell m}}{2}, \quad a_{\ell m}^B = i\frac{a_{2, \ell m} - a_{-2, \ell m}}{2}. \quad (4)$$

The $a_{\ell m}^E$ and $a_{\ell m}^B$ coefficients are then related to the E-mode and B-mode maps by

$$\begin{aligned} \tilde{E}(\hat{n}) &= \sum_{\ell m} \sqrt{\frac{(\ell+2)!}{(\ell-2)!}} a_{\ell m}^E Y_{\ell m}(\hat{n}), \\ \tilde{B}(\hat{n}) &= \sum_{\ell m} \sqrt{\frac{(\ell+2)!}{(\ell-2)!}} a_{\ell m}^B Y_{\ell m}(\hat{n}). \end{aligned} \quad (5)$$

It is important to note that the $a_{\ell m}^{E,B}$ coefficients are not exactly equal to the spherical harmonic decomposition coefficients of the E and B-modes maps. The use of the $a_{\ell m}^{E,B}$ coefficients without the factor $\sqrt{(\ell+2)!/(\ell-2)!}$ is partly a matter of convention. It is motivated by the fact that the power spectrum computed from such coefficients is the same on small scales as for the Stokes parameters themselves, while that of the \tilde{E} and \tilde{B} maps differs by a factor of $\propto \ell^4$. One consequence of the $\sqrt{(\ell+2)!/(\ell-2)!}$ factor is that white noise for the Stokes parameters anisotropy fields will transform to coloured noise for the \tilde{E} and \tilde{B} maps.

3 SIMULATIONS OF THE CMB POLARISED ANISOTROPY MAPS IN THE MULTI-CONNECTED UNIVERSE

To test the reliability of the codes used to search for the signature of a multi-connected topology in the Universe, we performed simulations of CMB skies for a flat universe with the topology of a 3-torus (Riazuelo et al. 2004). The first step in this process involved the computation of the coefficients of the E-modes, $a_{\ell m}^E$, in an analogous manner to the temperature anisotropy coefficients as described in Bielewicz & Banday (2011). The only significant difference was the use of the response function for the E-mode polarisation, $\Delta_\ell^E(k, \Delta\tau)$, instead of that for the temperature in their equation (1). The coefficients were then convolved with an appropriate beam profile, and subsequently transformed to the experimentally observed Stokes parameters using the relation (4) and inverse of the spin 2 spherical harmonics transform described in the previous section. Finally, homogeneous noise was added to the Q and U maps at a characteristic level for a given experiment.

In the simulations, we did not take into account the B-modes for multi-connected universes. The amplitude of the polarisation generated by gravitational waves is much smaller than the E-mode polarisation and the noise level for the Planck data is too high to allow the use of these maps for constraining the topology. We also do not consider effects that generate B-mode polarisation from the gravitational lensing of the E-modes. As is the case for any secondary effect, this dilutes signatures coming from the topology. Nevertheless, it is important for scales much smaller than those considered in this work ($\sim 20'$). However, it is important to note that in the simulated maps used in this paper B-modes are present due to the noise.

To study the signatures of a given topology, a CMB map is required with resolution comparable to the angular size of the first significant acoustic peaks of the polarisation map i.e. $\ell_{\max} \sim 400 - 500$. We have adopted $\ell_{\max} = 500$ in our simulations. The dimension of the fundamental domain of the 3-torus was $L = 2c/H_0$, which is about three times less than both the diameter of the observable Universe i.e. $\sim 6.6c/H_0$, and current lower bounds on the size of the Universe obtained from a matched circles study of temperature anisotropy maps (Key et al. 2007; Bielewicz & Banday 2011). Although such a small 3-torus does not fit very well to the data, the simulations are used only to demonstrate and compare the performance of the statistic for matched circles over a wide range of radii. A universe where there are many pairs of matched circles of different radii serves very well for these objectives. The values of the cosmological parameters adopted corresponded to the Λ CDM model (see Larson et al. 2011, Table 3) determined from the 7-year WMAP results. The time needed for the generation of one such CMB map on a single processor with clock speed 3 GHz is about 42 hours.

4 STATISTIC FOR THE MATCHED CIRCLES

If light had sufficient time to cross the fundamental cell, an observer would see multiple copies of a single astronomical object. To have the best chance of seeing ‘around the universe’ we should look for multiple images of distant objects. Searching for multiple images of the last scattering surface is then a powerful way to constrain topology. Because the surface of last scattering is a sphere centred on the observer, each copy of the observer will come with a copy of the last scattering surface, and if the copies are separated by a distance less than the diameter of the last scattering surface, then they will intersect along circles. These are visible by both copies of the observer, but from opposite sides. The two copies are really one observer so if space is sufficiently small, the CMB radiation from the last scattering surface will contain a pattern of hot and cold spots that match around the circles.

The idea of using such circles to study topology is due to Cornish, Spergel, & Starkman (1998). Therein, a statistical tool was developed to detect correlated circles in all sky maps of the CMB anisotropy – the circle comparison statistic

$$S_{p,r}^{\pm}(\alpha, \phi_*) = \frac{\langle 2 X_p(\pm\phi) X_r(\phi + \phi_*) \rangle}{\langle X_p(\phi)^2 + X_r(\phi)^2 \rangle}, \quad (6)$$

¹ because the measured Stokes parameters are independent of time we do not have time dependent terms in this case

where $\langle \rangle = \int_0^{2\pi} d\phi$ and $X_p(\pm\phi)$, $X_r(\phi + \phi_*)$ are temperature (or polarisation) fluctuations around two circles of angular radius α centered at different points, p and r , on the sky with relative phase ϕ_* . The sign \pm depends on whether the points along both circles are ordered in a clockwise direction (phased, sign $+$) or alternately whether along one of the circles the points are ordered in an anti-clockwise direction (anti-phased, sign $-$). This allows the detection of both orientable and non-orientable topologies. For orientable topologies the matched circles have anti-phased correlations while for non-orientable topologies they have a mixture of anti-phased and phased correlations. The statistic has a range over the interval $[-1, 1]$. Circles that are perfectly correlated or anticorrelated have $S = 1$ or $S = -1$, respectively, while uncorrelated circles will have a mean value of $S = 0$. To find correlated circles for each radius α , the maximum value $S_{\max}^{\pm}(\alpha) = \max_{p,r,\phi_*} S_{p,r}^{\pm}(\alpha, \phi_*)$ is determined. In case of anticorrelated circles the maximum value of $-S_{p,r}^{\pm}(\alpha, \phi_*)$ is used.

In the original paper by Cornish, Spergel, & Starkman (1998) the above statistic was applied exclusively to temperature anisotropy maps. However, it can also be applied to polarisation data and in this work, we focus on its application to the E-mode map. In this case, the X map is simply the map of the E-mode.

In order to speed up the computations, one can use the fast Fourier transform (FFT) along the circles, $X_p(\phi) = \sum_m X_{p,m} \exp(im\phi)$, and, by rewriting the statistic as $S_{p,r}^{\pm}(\alpha, \phi_*) = \sum_m s_m \exp(-im\phi_*)$, where $s_m = 2 \sum_n X_{p,m} X_{r,n}^* / \sum_n (|X_{p,n}|^2 + |X_{r,n}|^2)$, use the inverse fast Fourier transform. It reduces the computational cost $S_{p,r}^{\pm}(\alpha, \phi_*)$ from $\sim N$ to $\sim N^{1/2} \log N$ operations, where N is the number of pixels in the map.

In our studies we will use the statistic as modified in Cornish et al. (2004) to include weighting of the m th harmonic of the temperature around the p th circle $X_{p,m}$ by the factor $|m|$ such that the number of degrees of freedom per mode are taken into account.

$$S_{p,r}^+(\alpha, \phi_*) = \frac{2 \sum_m |m| X_{p,m} X_{r,m}^* e^{-im\phi_*}}{\sum_n |n| (|X_{p,n}|^2 + |X_{r,n}|^2)}, \quad (7)$$

where the Fourier coefficients $X_{p,m}$ are complex conjugated in the case of the $S_{p,r}^-$ statistic. Such weighting enhances the contribution of small scale structure relative to the large scales fluctuations. It is especially important because the large scale fluctuations, which are dominated by the integrated Sachs-Wolfe (ISW) effect for the temperature anisotropy and the reionisation effect in the case of polarisation, can obscure the image of the last scattering surface and reduce the ability to recognise possible matched patterns on it.

The general search explores a six dimensional space: the location of the first circle centre, (θ_p, ϕ_p) , the location of the second circle centre, (θ_r, ϕ_r) , the angular radius of the circle α , and the relative phase of the two circles ϕ_* . The number of operations needed to search for circle pairs scales as $\sim N^3 \log N$; thus implementation of a matched circles test is computationally very intensive. As a consequence, we restrict our analysis to a search for pairs of circles centered around antipodal points, so-called back-to-back circles. Then, the search space can be reduced to four dimensions and the number of operations to $\sim N^2 \log N$. Although the

back-to-back search can detect a large class of the topologies that we might hope to find, there remain those topologies that predict matched circles without antipodal configurations such as the Hantzsche-Wendt space for flat universes or the Picard space for hyperbolic universes (Aurich et al. 2004).

As in Bielewicz & Banday (2011), we used the HEALPIX² scheme with a resolution parameter $N_{\text{side}} = 512$ to define our search grid on the sky. To accommodate the use of the FFT approach, $n = 2^{r+1}$ points are used for each circle, where r corresponds to the resolution parameter of the map given by $N_{\text{side}} = 2^r$. By definition this is also the angular resolution used to step through ϕ_* . For α , we used steps a little bit smaller than the characteristic scale θ_c of coherent structures in the map, i.e. $2\theta_c/3$. As we discuss below, to increase the signal-to-noise ratio the polarisation maps were smoothed with a gaussian beam profile with FWHM of $50'$. Then, the scale of coherence is defined relative to the two-point correlation function as twice its angular width at half the maximum of the zero lag value. For polarisation maps this is around $20'$. Contrary to the temperature anisotropy this scale is smaller than the smoothing scale due to the bluer power spectrum, dominated by small angular scales, of the polarisation maps. The values of the E-mode map at each point along the circle were interpolated based on values for the four nearest pixels with weights inversely proportional to the distances between a given point and the pixel centers. Other methods of interpolation can be used but we have verified that even using the value of the nearest pixel does not change the value of the statistic significantly. The time needed for a search of one map using this scheme on a single processor with clock speed 3 GHz is about 180 hours.

To draw any conclusions from an analysis based on the statistic $S_{\max}^{\pm}(\alpha)$ it is important to correctly estimate the threshold for a statistically significant match of the circle pairs. The chance of random matches is inversely proportional to the number of coherent structures along the circles, therefore a false positive signal level of the statistic is especially large for circles with smaller radii. We used simulations of the maps with the same noise properties and smoothing scales as the Planck data (Planck Collaboration 2005) to establish the threshold such that fewer than 1 in 100 simulations would yield a false event.

It is important to note that the false detection level is smaller for higher resolution maps. Conversely, as shown by Cornish, Spergel, & Starkman (1998), the value of the statistic for matched circles

$$S_{\max} \approx \frac{\xi^2}{1 + \xi^2}, \quad (8)$$

depends on the ratio of the signal rms σ_s to the noise rms σ_n ratio, denoted ξ for the map, thus the efficiency of the statistic to detect matched circles is increased by smoothing the data. A smoothing scale should therefore be adopted that is a reasonable trade-off between these two requirements.

To eliminate the regions most contaminated by Galactic foreground residuals, we utilise the polarisation analysis mask P06 defined by the *WMAP* team (Page et al. 2007).

² <http://healpix.jpl.nasa.gov>

However, the statistic is very sensitive to the masking, particularly if a significant fraction of one or both circle pairs lies in the masked region. In this case, there is a significant probability to find a chance correlation of the temperature pattern between unmasked parts of the circles, thus increasing the false detection level. The effect is most pronounced for circles with the largest radii, close to 90 degrees, as well as for very small radii. To avoid this, we restrict our analysis to those pairs for which less than half the length of each circle is masked. Although the statistic computed in this way is not optimal, it is much more robust with respect to masking.

5 DISCUSSION OF DEGRADING EFFECTS FOR TEMPERATURE MAPS

Hitherto, the search for matched circles was performed only for the CMB temperature anisotropy maps. The main obstacle to the application of this method to polarisation data is the very high noise level of the currently available CMB polarisation maps. This situation will change with the release of the Planck data. As we will show below, in this case the typical noise amplitude will be sufficiently low to enable the use of the polarisation data in studies of topology. This provides an opportunity to avoid some of the problems inherent in the analysis of temperature maps.

Since the signatures of topology are imprinted on the surface of last scattering, any effects that dilute this image will also degrade the ability to detect such signatures by means of the matched circles statistic. In the case of the temperature fluctuations, there are two sources of anisotropy generated at the LSS: the combination of the internal photon density fluctuations and the SW effect, described by the first term of equation (1), and the Doppler effect, described by the second term. In the latter case, the correlations for pairs of matched circles can be negative in universes with multi-connected topology. As we can see in Fig. 1, for pairs of back-to-back circles with a radius smaller than 45° the dot products $\hat{n} \cdot \mathbf{v}$ have opposite signs for two nearly antipodal directions of observations. The Doppler term becomes increasingly anticorrelated for circles with smaller radius. One can show that on average in a flat universe with orientable topology the statistic (6) for this term is $\langle S^-(\alpha) \rangle = -\cos(2\alpha)$ (Riazuelo et al. 2006). Anticorrelation of the Doppler term significantly decreases correlations with the circle radius for the total CMB anisotropy map. We can see this effect in Fig. 2 for a simulated CMB sky in a universe with the topology of the 3-torus.

The ISW effect, as for any other secondary anisotropy contribution, will also smear out signal coming from the LSS. However, because it dominates only at large angular scales, this can be remedied either by high-pass filtering of the map or by using the matched circles statistic with m weighting (7) to prevent the statistic from being dominated by the large angular scales.

The consequences of these degrading effects are weaker constraints on the topology of the Universe obtained from the matched circle statistic. As we can see in Fig. 2, use of the CMB map without both of the degrading effects would allow us to impose lower bounds on the minimum radius of the correlated circles which can be detected much below the present constraints $\alpha_{\min} \approx 10^\circ$ (Bielewicz & Banday 2011).

6 SEARCH OF MATCHED CIRCLES IN POLARISATION MAPS

As already mentioned in Section 2.2, a polarisation map at small angular scales can be considered as a snapshot of the LSS. Theoretically, then, the polarisation provides a better opportunity for the detection of multi-connected topology signatures than a temperature anisotropy map. The only serious issues preventing its use in studies of topology are instrumental noise and the correction of the polarised data for the Galactic foreground. Since the polarisation anisotropy amplitude is around ten times smaller than the amplitude for temperature, these constitute more serious problems than for temperature anisotropy maps.

In addition, since polarisation is generated by the scattering of a quadrupolar distribution of radiation on free electrons at the moment of recombination or during reionisation, the orientation of the local quadrupole can be a significant problem to this analysis. The orientation of the dipole and quadrupole are determined uniquely by the initial density field. Thus, one can see from equation (2) that the generated polarisation depends on the projection of the quadrupole onto the plane perpendicular to the line of sight. As shown by Riazuelo et al. (2006) this projection can be expressed by $1 - (\hat{n} \cdot \hat{\mathbf{v}})^2$. This is analogous to the projection of the dipole direction on the line of sight for the Doppler term, as discussed in the previous section. Clearly, the contribution is included in our simulations, and any deleterious effect is accounted for in the statistical analysis.

The other effects that might degrade the signal from the LSS are the polarisation generated after reionisation and the gravitational lensing effect. The former prevails at very large angular scales where, as for the ISW effect and temperature anisotropy data, its influence can be eliminated by either high-pass filtering or the use of the statistic (7). We will investigate this below in subsection 6.1. The lensing effect is most significant at very small scales ($\sim 3'$) and is negligible for the range of scales of interests in this paper.

We have tested the performance of the matched circles statistic for simulated Planck (Planck Collaboration 2005) and CORe (CORe Collaboration 2011) polarisation maps. Although the latter has not currently been selected as a candidate mission by ESA, we will consider it as a useful reference mission for the next generation of observational CMB projects.

Before we present the results for simulations, it is worth considering a rough estimation of the performance of the matched circle statistic for different experiments. As we have already mentioned, the noise level of the *WMAP* polarisation data is too high to enable detection of the matched circles. Even for the map assembled from the *WMAP* Q, V and W-band polarised data with inverse noise weighting we find $S_{\max} \approx 0.05$. This is considerably below the false detection threshold of the statistic. There is a much improved prospect for the detection of the signatures of topology for the Planck data. As for the *WMAP* data, to decrease the noise level we consider the map obtained from a combination of the data with the lowest noise levels using inverse noise variance weighting. Using the sensitivity and resolution of the polarised anisotropy map derived from the 100, 143 and 217 GHz frequency maps, i.e. with $\Delta_N^{Q,U} \approx 59 \mu\text{K arcmin}$ and $\theta_{\text{FWHM}} = 10 \text{ arcmin}$ (Planck Collaboration 2005), re-

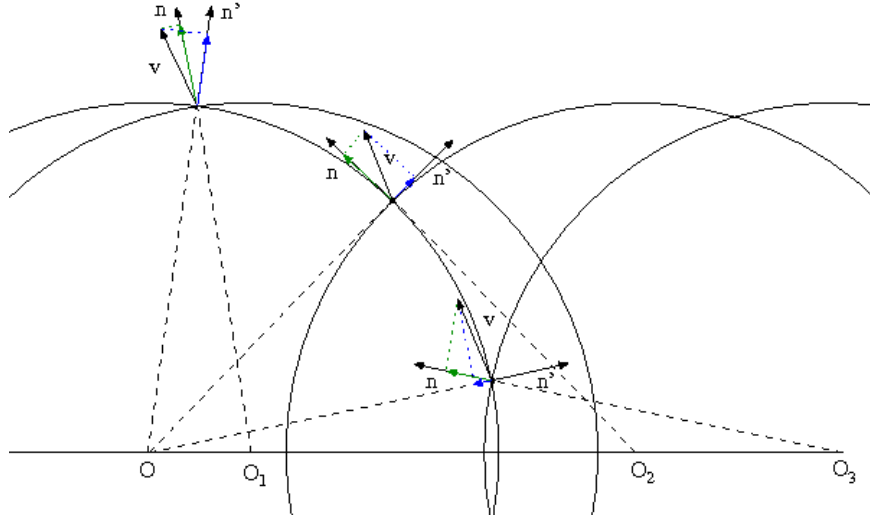


Figure 1. The baryon velocity \mathbf{v} at the last scattering surface seen from two different directions \mathbf{n}, \mathbf{n}' in the multi-connected universe. For the radius of the matched circles larger than 45° the projections of the baryon velocity on the direction of observations (dashed lines) for the observers O and O_1 are strongly correlated. For the radius equal to 45° (observers O and O_2) the velocity projections are independent and for the radius smaller than 45° (observers O and O_3) the projections are anti-correlated. The projections are denoted by blue and green arrows.

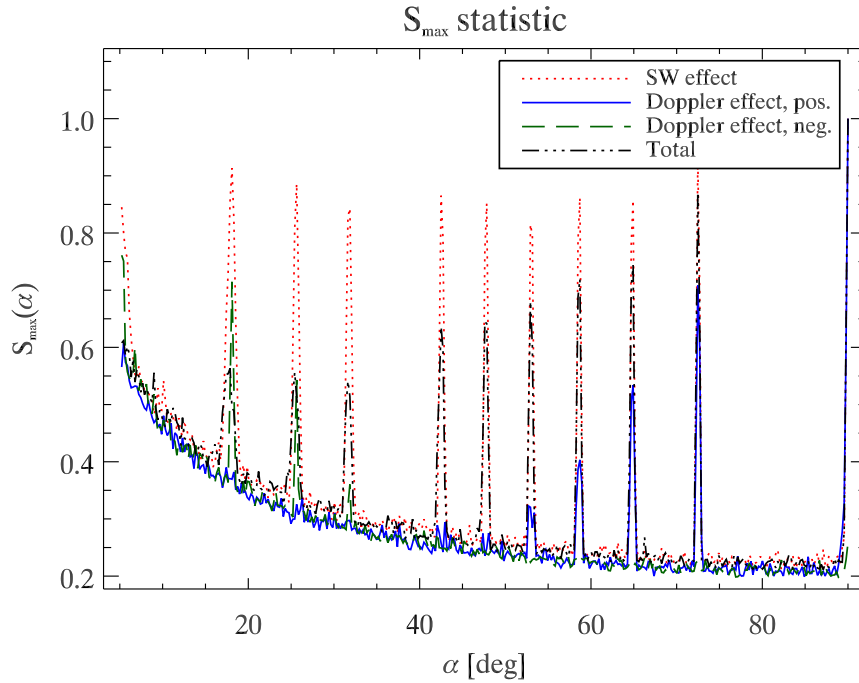


Figure 2. An example of the S_{\max}^- statistic for a simulated CMB temperature anisotropy map of universe with the topology of a cubic 3-torus with dimensions $L = 2 c/H_0$. The dotted, solid, dashed and three dot-dashed lines show the statistic for CMB maps of the Sachs-Wolfe (SW) effect, the positive and negative correlations of the Doppler effect and total anisotropy, respectively.

spectively, we get $S_{\max} \approx 0.6$. This is sufficient to enable the detection of pairs of matched circles. Conversely, the noise level for single frequency maps seems to be too large to allow such a test – even the 143 GHz channel with a sensitivity $\Delta_N^{Q,U} \approx 82 \mu\text{K arcmin}$ (Planck Collaboration 2005) only renders a value of $S_{\max} \approx 0.4$. The best chance of detection for such signatures of topology arise for an experiment

with specifications similar to those of the CoRE project. For a channel with minimal contamination by the Galactic foreground, i.e. 105 GHz, with a polarisation sensitivity of $\Delta_N^{Q,U} \approx 4.6 \mu\text{K arcmin}$ (CoRE Collaboration 2011), the statistic $S_{\max} \approx 0.99$ achieves a value close to the maximum.

As described in Section 3, we simulated the CMB polarisation maps, in terms of the observable Stokes parameters

Q and U , for a universe with the topology of a 3-torus of size $L = 2c/H_0$. The resolution of the maps and the noise rms correspond to the Planck 100, 143 and 217 GHz frequency maps coadded with inverse noise weighting and the CORe 105 GHz frequency map. In both cases, the maps were convolved with a gaussian beam of $\text{FWHM}=10'$ and homogeneous noise with a sensitivity of $\Delta_N^{Q,U} \approx 59 \mu\text{K arcmin}$ and $\Delta_N^{Q,U} \approx 4.6 \mu\text{K arcmin}$, respectively, were added.

Since the direct use of the Stokes parameters in the analysis would require the rotation of values into a common coordinate system (defined by the pairs of points in question) which can significantly slow down the computations, the analysis was carried out for E-mode maps obtained from the Q, U maps by means of the relation (4). The use of the spin 0 maps allows us to speed up computations by the application of an FFT along the circles, as described in Section 4. The maps were also smoothed with a beam of $\text{FWHM}=50'$ to increase the signal to noise ratio. Because of the very blue power spectrum of the E-mode maps, caused by the $\sqrt{(\ell+2)!/(\ell-2)!}$ factor (see equation (5)), in order to remove the noise on the smallest angular scales the width of the beam profile has to be wider than would be the case for the temperature map. The smoothing serves also to eliminate from the analysis multipoles of order larger than the maximum order used for the simulated CMB maps i.e. $\ell_{\text{max}} = 500$.

The S_{max}^- statistic for the simulated maps is shown in Fig. 3. As expected the amplitudes of the peaks do not decrease with the radius of the circles as in the case of the temperature anisotropy maps. We see that pairs of matched circles can be detected for the Planck coadded map. However, comparing with Fig. 2 one should notice that the relative amplitude of the peaks with respect to the average correlation level for the circles with small radius is not bigger than for the temperature map. Thus, the constraints on topology will not be much tighter than those derived from an analysis of temperature maps. A much better perspective arises for the CORe maps. The signal of the multi-connected topologies is very pronounced in this case enabling the detection of matched circles with very small radius. It is also worth noticing that the width of the peaks is smaller than for temperature maps. It is a consequence of domination of the small angular scales in the power spectrum of the polarised maps leading to more narrow peak of zero lag of the two-point correlation function.

To determine more precisely the minimum radius of the detectable circles it is needed to estimate the false detection level of the statistic. We used for this purpose 100 simulations of the Planck coadded map for a universe with a simply-connected topology. The detection level was estimated for both the full sky Q and U maps and the masked maps. The former corresponds to the case of a perfect separation of the CMB and the Galactic foreground polarisation and yields optimal performance of the statistic. The latter presents a more realistic estimation taking into account the necessity of removing those regions most contaminated by the Galactic foregrounds.

We used masks based on the P06 mask derived by the WMAP team (Page et al. 2007) for their analysis of polarised data. In order to minimize ambiguous modes that arise from the decomposition of the Stokes parameters into the E and B-mode on incomplete sky coverage, we used the

following procedure for computation of the E-mode maps. The P06 mask was extended and apodized employing the method described by Kim (2011) (referred to as a 'processing mask' therein) i.e. the mask was widened so that after apodization with a Gaussian profile it is zero in regions defined as such by the original mask up to a precision of 10^{-6} . Then, the E/B decomposition of the Q and U maps was performed with this mask. As for a full sky analysis, in order to increase the signal to noise ratio, the E-mode maps were also smoothed with a beam of $\text{FWHM}=50'$. Due to the apodization, most of the ambiguous modes are localized close to the mask edges. They can then be eliminated from the computation of the statistic by use of an additional mask applied to the derived E-mode map, but further broadened then apodized in a similar fashion to the previous mask. In this case, apodization helps to minimize the ringing effect which might appear on account of the use of the FFT in computations of the matched circles statistic. In both cases we used apodization with a Gaussian of $\text{FWHM}=60'$. Then, the fraction of the unmasked part of the sky for the cut employed in the computation of the statistic is $f_{\text{sky}} = 0.55$. It is much less than for the original mask with the fraction $f_{\text{sky}} = 0.73$. The masks are shown in Fig. 4. Such an aggressive excision of data corresponds to a rather extreme case. We may hope that the broader frequency range and lower noise level of the Planck data will enable us to significantly extend the useful sky coverage for cosmological analysis.

This procedure for the computation of the E-mode signal does not remove completely leakage from the B-modes. However, the residual is no larger than the noise level in smoothed polarisation maps that is negligible compare to the prevailing of E-mode signal. It is worth noticing that in our procedure the residual signal from B-modes is additionally damped by smoothing.

The false detection levels established from the requirement that fewer than 1 in 100 simulations should yield a false match are shown in Fig. 3 corresponding to two extreme cases: analysis on the full sky and on a map with a very conservative mask. The detection threshold for both Planck and CORe should lie somewhere between these two curves. However, more detailed studies of the influence of the Galactic foreground polarisation is required, but is beyond the scope of this paper.

The minimum radius, α_{min} , of the correlated circles which can be detected for a given map is determined by the intersection of the peaks in the matching statistic with the false detection level. The height of the peaks indicates that the minimum radius is around 20° for the Planck and around 5° for the CORe data in the case of the masked map. If we do not detect any pairs of matched circles, this radius will determine a lower bound on the size of the fundamental domain.

6.1 Impact of reionisation on the matched circles statistic

One of the secondary anisotropy effects which might degrade our ability to detect pairs of matched circles in polarisation induced by the Thomson scattering of the CMB photons on the reionised matter. To evaluate the level of impact of the effect on the performance our statistic, we simulated the CMB map for the multi-connected universe both with

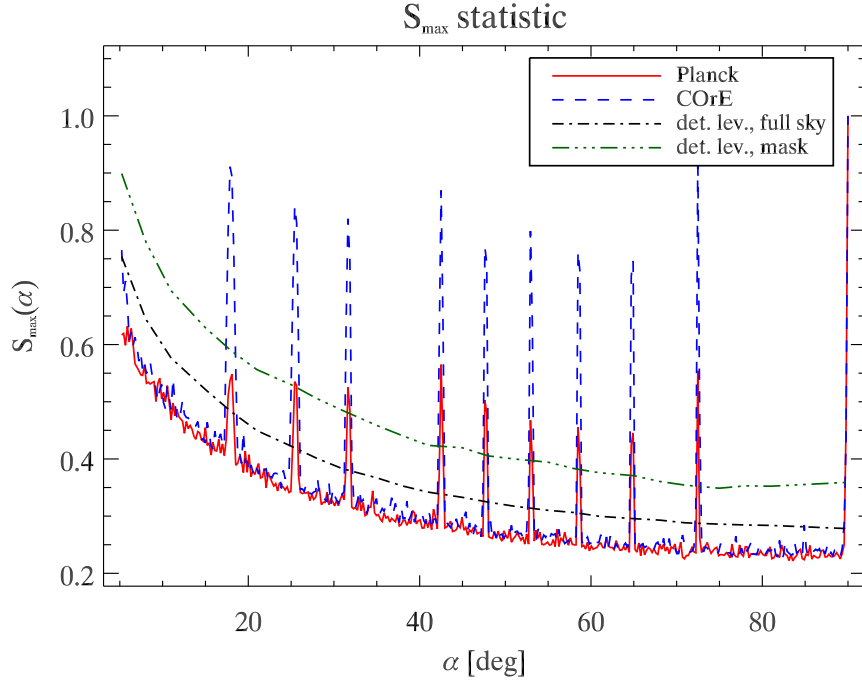


Figure 3. An example of the S_{\max}^- statistic for a simulated polarised CMB map of the universe with the topology of a cubic 3-torus of dimension $L = 2 c/H_0$. The solid and dashed lines show the statistic for simulated polarisation maps with angular resolution and noise level corresponding to the Planck and COrE data, respectively. The dot-dashed and three dot-dashed lines show the false detection levels for the statistic estimated from 100 Monte Carlo simulations of the Planck coadded 100, 143 and 217 GHz frequency polarisation maps for the full sky and cut sky analysis, respectively.

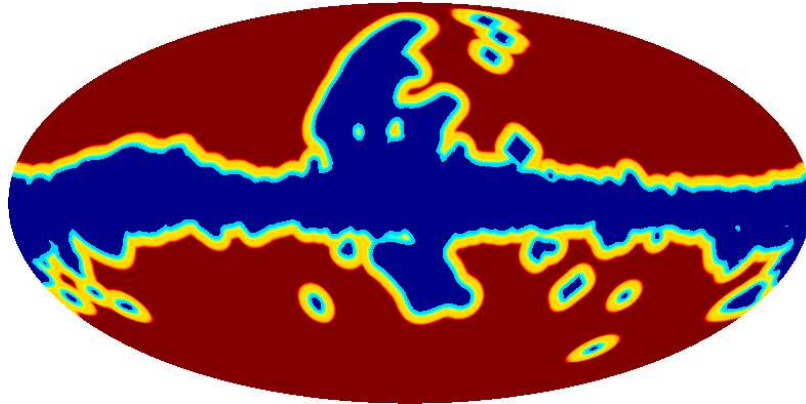


Figure 4. Masks used in the analysis. The dark blue region corresponds to the part of the sky unused in polarisation analysis, defined by the *WMAP* team in the P06 mask. The light blue and yellow bands correspond to the apodized regions of the extended mask used for the E/B decomposition and the mask used in the search for the matched circles, respectively. The red region corresponds to the unmasked and unsmoothed part of the sky.

and without reionisation and compared the statistics. The optical depth for the reionisation effect was set as $\kappa = 0.088$. This comparison showed that the relative difference between the statistics is of order of 10^{-3} . Therefore, the influence of polarisation generated after reionisation is negligible for the detectability of the matched circles. This should not be a surprising conclusion since the S_{\max} statistic is sensitive rather to small angular scales while the polarisation effect of reionisation prevails at large scales. In case of the E-mode map the small scales are also additionally amplified by the

$\sqrt{(\ell+2)!/(\ell-2)!}$ factor, thus the large angular scales are not so significant as for the temperature anisotropy.

7 CONCLUSIONS

We have studied the possibility of using CMB polarisation maps for constraining the topology of the Universe by means of the matched circles statistic. We have shown that the CMB polarisation maps are free from the degrading effect of the Doppler term that contributes to the temperature

anisotropy and that consequently allows matched circles to be detected with a radius much smaller than for the CMB temperature maps thus providing tighter constraints on the topology. Moreover, the detection is robust with respect to the secondary CMB polarisation generated after reionisation. However, the application of polarisation data to studies of the topology of the Universe depends to a large extent on systematic effects such as noise and Galactic foreground residuals.

Using simulations of the Planck polarisation data we demonstrated that the level of the instrumental noise should be sufficiently low to enable the detection of the signatures of a multi-connected topology, though it will not be low enough to improve constraints on the topology derived from the temperature maps. Nevertheless, it can serve as an important cross-check of the latter. The best chance for the detection of the matched circles is afforded by a map obtained from the inverse noise weighted combination of the three least noisy Planck frequency maps i.e. 100, 143 and 217 GHz maps. The polarised Galactic emission should also be the weakest for the first two of these channels enabling the best estimation of the CMB polarisation.

All the advantages of the polarisation analysis can be fully exploited in the case of the planned next generation of full sky low-noise experiments. To demonstrate this, we used simulations with the resolution and sensitivity corresponding to the CORe mission. In this case the smallest radius of the detectable back-to-back matched circles in the polarisation maps is significantly smaller than for the temperature maps. An estimation based on our simulations shows that the radius is around 5° .

The detectability of the matched circles depends heavily on our ability to reliably separate the CMB polarisation from the polarised emission of the Galaxy. We restricted our studies on this issue only to the application of a mask intended to remove those areas of the sky most contaminated by the Galactic foreground emission. We assumed that the residuals of the Galactic emission outside the mask can be sufficiently well modelled and subtracted from the map. However, it is not certain to what extent this will be possible. As is well known, removal of the polarised Galactic emission is a more challenging problem than for the temperature maps since there is no frequency band where the CMB polarisation dominates over the polarisation of the Galactic foreground even for a small fraction of the sky.

We also have to bear in mind that a search for matched circles on a masked map is able to constrain only those universes with such dimensions and orientation of the fundamental domain with respect to the mask that allow the detection of pairs of matched circles. Since the probability of overlooking circle pairs may be significant for a mask that removes a large fraction of the sky, employing a cut as small as possible is one of the crucial issues. The analysis of a polarised version of an internal linear combination-type (ILC) map on the full sky is also unlikely to be a credible solution of this problem. As shown in Bielewicz & Banday (2011) for the temperature case, Galactic foreground residuals in the Galactic plane of an ILC map can cause significant spurious correlations between the circles. In the case of polarisation maps it will probably be an even more pronounced effect. However, it is to be hoped that the high quality polarisation data from the Planck satellite will enable a better under-

standing of the Galactic polarised emission and therefore the use of a larger fraction of the sky for cosmological analysis than for the *WMAP* data.

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